1. (a) During the lecture we showed that $(S(t), W(t)) = (0, 0)$ is a fixed point of the system

$$\frac{d}{dt}S(t) = 2S(t) - S(t)W(t)$$
$$\frac{d}{dt}W(t) = -3W(t) + S(t)W(t),$$

find the second fixed point.

(b) Using the same approach as in lecture and in part (a), find all fixed points of the general predator-prey model

$$\frac{d}{dt}S(t) = r_1S(t) - k_1S(t)W(t)$$
$$\frac{d}{dt}W(t) = k_2S(t)W(t) - r_2W(t),$$

2. In this problem we vary some of the model parameters to see their impact on the seal and whale populations, $S(t)$ and $W(t)$.

(a) Begin by setting $r_1 = r_2 = k_1 = k_2 = 1$ with $S(0) = 2$ and $W(0) = 1$. Notice the largest and smallest values of both $S(t)$ and $W(t)$.

(b) Looking back to our construction of Model 3, what role does k_2 play? Keeping the same initial values, $S(0)$ and $W(0)$, set $k_2 = 1.5$ and run the code. Do the results match your intuition? Discuss the results amongst your group.

(c) Looking back to our construction of Model 3, what role does k_1 play? Keeping the same initial values, $S(0)$ and $W(0)$, set $k_1 = 1.5$ and run the code. Do the results match your intuition? Discuss the results with your group.

3. In your code, set $r_1 = 4, k_1 = 2, r_2 = 5$ and $k_2 = 2$. This yields the model

$$\frac{d}{dt}S(t) = 4S(t) - 2S(t)W(t)$$
$$\frac{d}{dt}W(t) = 2S(t)W(t) - 5W(t).$$

(a) Choosing $S(0) = 1$ and $W(0) = 2$, verify mathematically that $\frac{d}{dt}S(t) = 0$. Now run the code. Explain why $S(t)$ begins to change despite $\frac{d}{dt}S(t) = 0$ initially.

(b) In part (a) we saw $S(t)$ begin to change despite $\frac{d}{dt}S(t) = 0$ initially. When it began to change, did it increase or decrease? Can you explain this mathematically?

(c) Can you find initial conditions $S(0)$ and $W(0)$ which cause $S(t)$ and $W(t)$ to both be decreasing initially? Can you make them both increase initially? Verify your calculations with the code.