

Manipulating Equations

$$\begin{aligned} 2 \times 3 &= 6 \\ \frac{2 \times 3}{3} &= \frac{6}{3} && \text{cancel the threes on the left} \\ 2 &= 2 \end{aligned}$$

Quantity a times quantity b , or $a \times b$, is often written just as ab .

$$\begin{aligned} ab &= c \\ \frac{ab}{b} &= \frac{c}{b} && \text{cancel each } b \text{ on the left} \\ a &= \frac{c}{b} \end{aligned}$$

$$\begin{aligned} \frac{a}{b} &= c \\ \frac{a}{b}b &= cb \\ a &= cb \end{aligned}$$

$$K = \frac{1}{2}mv^2 \quad \text{kinetic energy}$$

$$U = -\frac{GMm}{r} \quad \text{potential energy}$$

$$E = k + U \quad \text{energy}$$

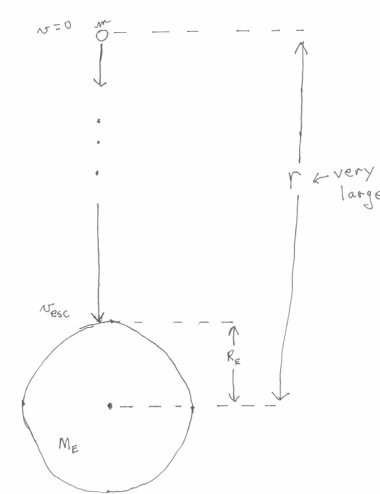
$$c = 2.998 \times 10^8 \text{ m/s} \quad \text{speed of light}$$

$$G = 6.67 \times 10^{-11} \quad \text{gravitational constant}$$

$$M_E = 5.974 \times 10^{24} \text{ kg} \quad \text{Earth's mass}$$

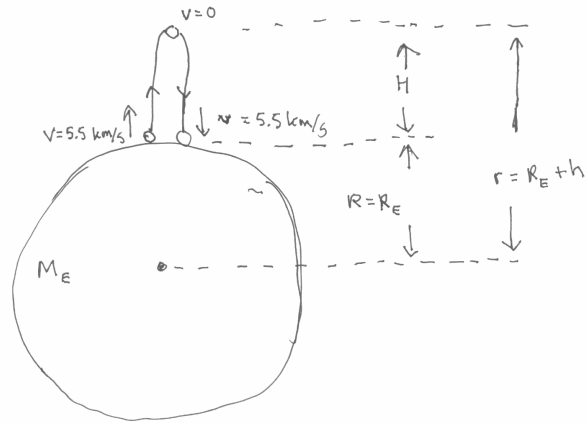
$$R_E = 6371 \text{ km} \quad \text{Earth's radius}$$

Problem 1 Use energy to find the escape speed of the Earth. Use $R = R_E =$ radius of Earth and $M = M_E =$ mass of Earth.



- When the object is at rest very far from the Earth, what is its kinetic energy K ?
- When the object is at rest very far from the Earth, what is its potential energy U ? Think about this.
- Find an expression for the object's energy, $E_2 =$ kinetic energy + potential energy, when it is at rest very far from the Earth.
- Find an expression for the object's kinetic energy when it gets to the Earth. Express your answer in terms of m and v_{esc} .
- Find an expression for the object's potential energy when it gets to the Earth. Express your answer in terms of M_E , m , G , and R_E .
- Find an expression for the object's energy, $E_1 =$ kinetic energy + potential energy, when it reaches the Earth's surface.
- The object's energy doesn't change as it falls, so set $E_1 = E_2$, and find the escape speed v_{esc} .

Problem 2. Use energy to find the maximum height h when



- Find an expression for the energy, $E_1 = \text{kinetic energy} + \text{potential energy}$, at launch.
- Find an expression for the energy, $E_2 = \text{kinetic energy} + \text{potential energy}$, at maximum r .
- These two energies are equal, i.e., set $E_1 = E_2$, and cancel m from each of the terms.
- Solve for the height h by Using $r = R_E + h$.
- Find the height h .